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Education and Training

PRINCIPLES AND METHODS OF TEACHING ARITHMETIC TO MENTALLY RETARDED CHILDREN *

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IT IS generally agreed that experience and intuitive insight enables the gifted teacher to find productive ways of coping with the defective mind. There are, however, definite limitations to such an intuitive approach . . . limitations that may be gradually overcome by carefully planned experimentation into the processes of learning. The child's understanding of number operations rather than his attainment of mere mechanical skill should be the educational goal considered in research in arithmetic.[1]

Research into adequate teaching methods demands (1) a thorough knowledge of the normal development of number concepts, (2) an analysis of the general and specific deficiencies which cause retardation of this development, and (3) the tryout of various educational principles and devices aimed at the removal of the difficulties in the process of learning.

Basically, the use of a number concept depends on an understanding of the number system. The child must have a spatial reference pattern in which the numbers are related; in the series 1-2-3-4-5-6-7-8-9-10 he must know that every number has a definite position with respect to every other number. He must learn that the entire number plan is based on this order of ten, and, for example, that eighty means that eight groups of ten are involved.

The normal child has a gradual development in his understanding of number.[6] Many times the primary form of the number concept starts with body activity, motor rhythm, counting by fingers, etc. Concrete number forms are next used; these are perceptually organized units, especially within the visual field (for one example, the child is able to recognize this formation of dots ⁙ as 5). Finally, the picture-like references of the number forms are partially discarded, and the child reaches the level of abstract number concepts.

Since the educational process is a gradual development, any new step made by the child can only be reached if he has successfully passed through the preceding one. An important cause of the retardation of this development is the too rapid progress in the teaching of this subject. Particularly in the mentally retarded child his slow transfer from one successive step to the next often is not taken into account. A

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child may perform a mechanically advanced operation on the verbal level without an understanding of its concrete basis. Special deficiencies in those functions necessary for performance at the various achievement levels are another principal cause of disability in arithmetic.

For instance, as mentioned before, children may start to count by using their fingers. They may be impaired in this performance by a special deficiency in discriminating their fingers. We developed a test which is useful in the diagnosis of such a deficiency. The test measures the ability of the child to indicate with the eyes shut one or two fingers at a time which have been touched or named previously by the examiner. The results show a decrease in errors in the average performance on this test with increasing mental age. There is a relation between retardation on this test and arithmetic disability.[5, 8] (Fig. 1.)

A child's further progress depends on his ability to visualize concrete number represented as a configuration of elements. Several tests are used by us for diagnosing deficiencies of visual functions. One test measures the child's accuracy in the recognition of patterns of dots shown to him for a fraction of a second. A relationship exists between arithmetic disability and performance on this and other visual perception tests.[7, 9]

That the child first must think of number concretely before he uses it in an abstract verbal way is extensively demonstrated in the study made by Brownell.[2] He found that children who could not handle numbers in a concrete perceptual organization were retarded when they were required to handle numbers abstractly. These chil-

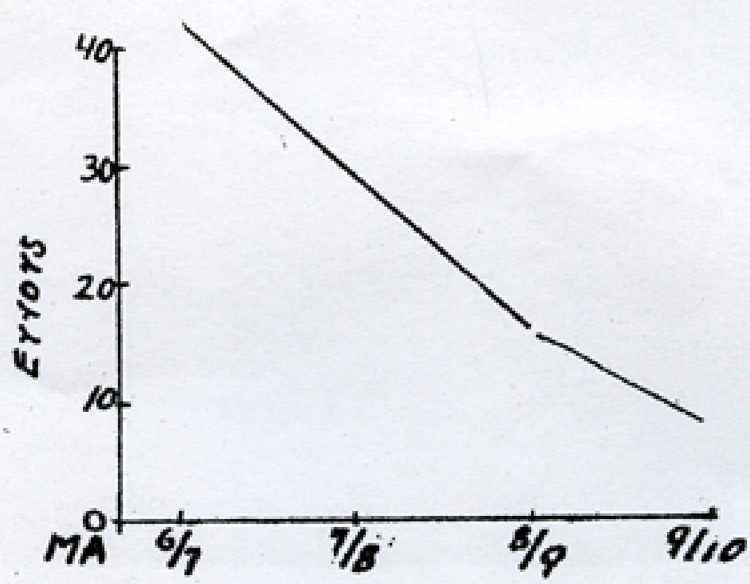


Fig. 1. Error Scores on Finger Schema Test

dren counted the concrete objects (which were in this case dots) one by one, and were not able to recognize that most numbers are an aggregate or unit made up of single parts.

Recently, a research program in the remedial teaching of arithmetic has been carried on at the Wayne County Training School, Northville, Michigan. A few principles underlying our teaching may be mentioned. The following two general principles deal with the spontaneity of the child toward number work:

1. The educational situation should offer many opportunities for the child's activity. Number operations should be concerned with doing instead of only seeing, hearing, or verbalizing.

2. Increased motivation is needed for children having special deficiencies. Games encouraging self- and group-competition should be widely used.

There are also two specific points which govern our methods in remedial number work:

1. The primary steps in number development must be taught very slowly. Each step can be divided into sub-steps so that the rigidity in the transfer known to be characteristic of the mentally retarded child can be overcome. The mentally retarded child, because of his mental make-up, must be kept much longer in the concrete field than the normal child.

2. The deficient function must be taken into consideration . . . it may be handled in one of the following ways:

a. Strengthen the deficient function by direct practice. For improving finger dexterity in number work Descoeudres recommends finger exercises for the mentally retarded child. The child is required to stretch successively 1, 2, 3, 4, 5, etc., fingers as they are called by the teacher. In cases of a deficiency in visual discrimination, we have the child practice the recognition of visual concrete configurations.

b. Imbed the deficient function in a more comprehensive activity which results in more efficient work. For instance, in cases of deficiencies of visual space relationships, the learning of the decimal system may be facilitated by additional kinesthetic activity (tapping, etc.).

c. In cases of severe functional disability, replace the deficient function by a function that can be efficiently used in the learning of arithmetic.

We may now discuss somewhat in detail the special methods and devices used in our remedial program. The children which were subjected to remedial treatment were all high-grade morons having various accomplishments in arithmetic, but all needing training at the primary levels in their understanding of number. All had individual disabilities pertaining to a deficiency of space relations, and, as a result, their concrete number concept was impaired. Therefore, the remedial work was directed toward an understanding of arithmetic operations on the basis of concrete number references.

The concrete number concepts were introduced in various ways. A pegboard four feet square is one of the first devices used (Fig. 2). By setting the board on the floor with the pegs vertically placed, games consisting of tossing rubber rings at the pegs may be devised. This board gives opportunity for counting and perceiving concrete units made up by the pegs; it may also be used later for addition, subtraction, and multiplication games.

The perception of a configuration of elements as number can be taught in various ways. By abstracting the child learns eventually to recognize a configuration of elements as representing a number. If, for instance, the child is able to match two patterns different in design but identical in number of elements, he has arrived at an understanding of concrete number. Decroly, Descoeudres, and others have successfully used a lotto game for this training; on the basis of number the child matches cards having variously configurated patterns.[3] We prefer the following method which calls for more activity on the part of the child than is required in pure matching. The child has before him a narrow strip of wood on which he is to place small chips in a series identical in number with a pattern on the peg-board (Fig. 2, bottom, left). Domino games in which the numbers are represented as patterns of dots or numbers or both serve the same purpose (Fig. 2, bottom, right). Games involving the throwing of dice may also be used for training in concrete numbers.

Several basic devices were constructed for introducing the child into the decimal number system. The peg-board (since it contains ten by ten holes) offers a first opportunity to get acquainted with the decimal system. A similar device which can be more easily handled is the marble-board (Fig. 3). It consists of a cardboard

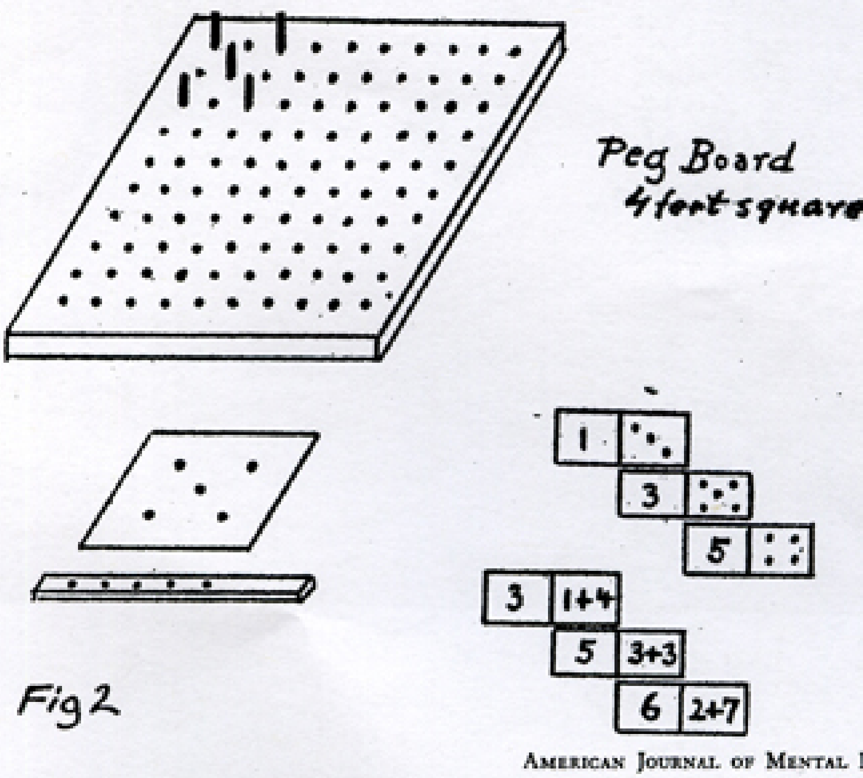


Fig 2

11 inches square with ten rows of ten holes each; marbles of various colors can be placed into the holes. The marble-board again has the advantage of tying up the child's movements with number operations in the visual field. Marbles can be actually removed or placed upon the board by the child in processes of addition or subtraction or they may remain on the board and a wooden pointer can be used for these operations. In this way the child learns to recognize number units and individual numbers as localized parts of a series within the decimal system. Fig. 3 shows the child's performance on the problem of ten take away two.

With some of our children particularly deficient in space relationships we found that units represented by color aided the child. For instance, the first fives or tens can be set up against the next fives or tens by variation of their colors.

Another universally used basic device is the abacus. The form which we used is represented in Fig. 3. This device does not possess the precise spatial positions of the marble-board but it has the advantage of facile operations. It consists of ten rows of ten wooden beads strung on wires and set into a wooden frame. Each row of ten was broken into two sets of five by inserting a small bead, contrasting in color to the others, between the two sets.

Addition and subtraction appear in the work on the abacus as two directions of movements, i.e. one to the right when taking away and the other to the left when adding on. It should be noted that addition and subtraction were taught simultaneously as two

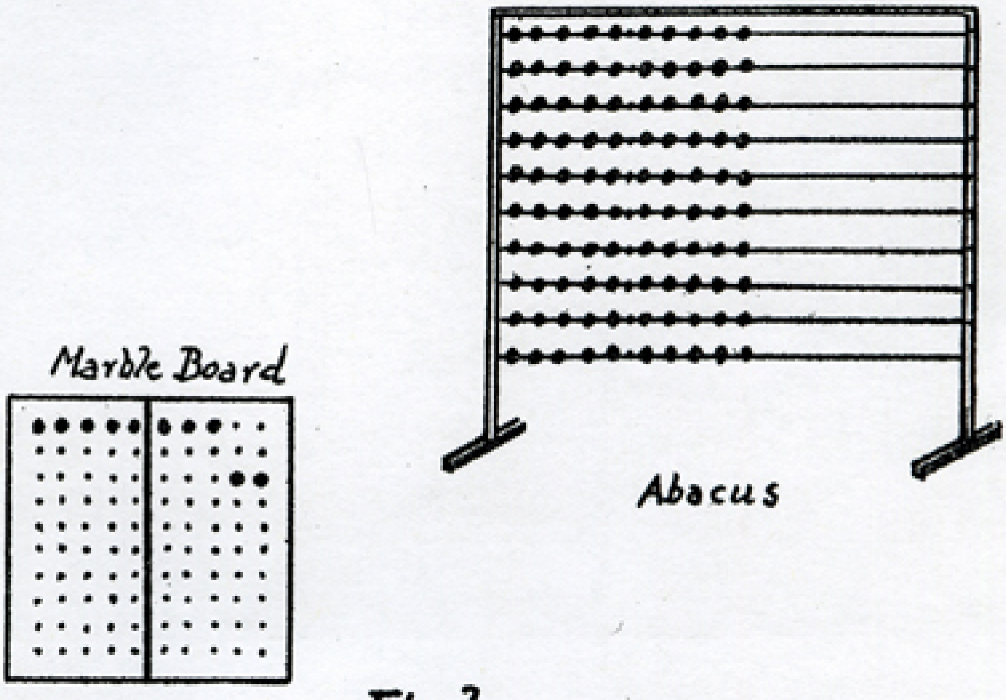
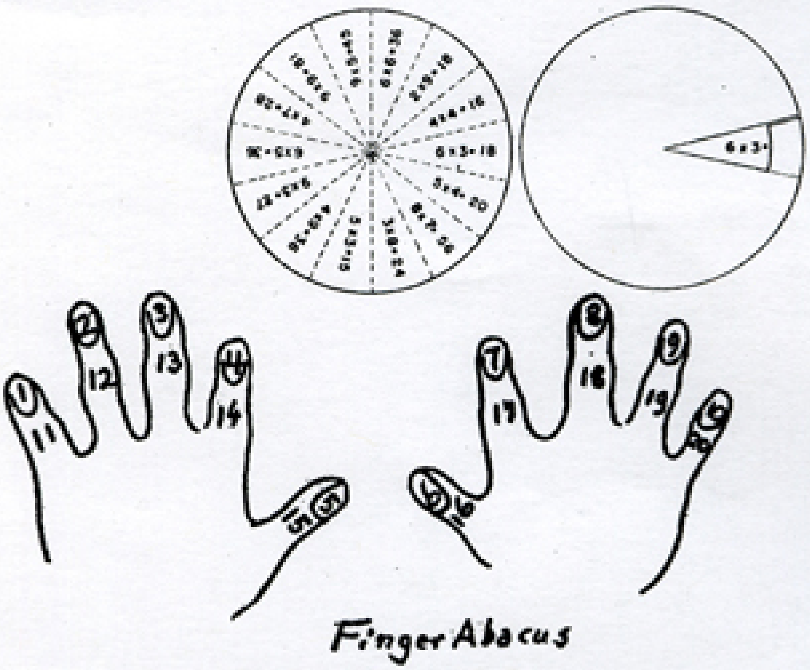


Fig. 3

phases of the same fundamental process. Occasionally, one finds a child who must remain longer on the more static devices such as the marble-board because of a left-right confusion and a lack in the discrimination of space relations.

Before proceeding to addition and subtraction with more than one row of ten the child must be acquainted with the basis significance of ten on the abacus. On the other hand, just this feature of the abacus may be a handicap for some children; particularly children of very rigid behavior (perseverative tendencies) may overlook the continuity of the number series in favor of single rows of ten. Therefore, other devices must be added. For instance, one device which we used consisted of a vertical row of numbered dots in a continuity of up to fifty or even more by means of which addition and subtraction can be executed.

For children who have a severe deficiency in visual space discrimination the primary work on the marble-board and on the abacus does not always give satisfactory results. Consequently, we introduced the finger-abacus. The child places his hands flat on the desk and uses them as a device in the following way (Fig. 4). The nails are counted as a consecutive series of numbers from one to ten and the knuckles from 11 to 20. The procedure of adding and taking away on the finger abacus is in principle the same as that of the peg-board, marble-board, and abacus. This finger-abacus adds a kinesthetic element to the visualization of concrete numbers. Moreover, it is always available for a check on abstract number operations, and, therefore, especially valuable



Finger Abacus

Fig. 4

for children who have great difficulty in abstract number work or who have poor memory.

Addition and subtraction problems are presented rather early as written number figures simultaneously with the work on the various concrete devices. Such difficult processes as borrowing and carrying are gradually introduced; they, too, are performed at first as concrete operations on the abacus. One must stay relatively long on the intermediary level of writing down explicitly the sucessive steps in these processes. For instance, in the subtraction problem of 14 from 22 it must be written for some time as:

$$\begin{array}{r} 20+2 \\ -10+4 \\ \hline \\ \end{array} \quad \text{or} \quad \begin{array}{r} 10+12 \\ -10+\ 4 \\ \hline 0+\ 8 \end{array} \quad \text{or} \quad \begin{array}{r} 22 \\ -14 \\ \hline 8 \end{array}$$

Multiplication and division are introduced as two related processes, and both must be first taught thoroughly on a concrete basis. The abacus is used to present multiplication as an outgrowth of addition. Children with severe defects in visualization find it impossible to understand or to remember even the simplest times facts without actual practice with a concrete representation. Even later when multiplications are verbalized the child is at first required to check each problem by either concrete operations on one of the devices or the additive verbal method. Here again games may be introduced. For instance, bingo may be used as a multiplication game by placing multiplication problems on the cards that are called. As mentioned before, the peg-board offers many possibilities for multiplication practice. Another device which presents an opportunity for self- and group-competitive games is the multiplication wheels. Each wheel consists of two paper disks. The lower one is marked into sections into each of which the successive times facts are written; a slit is cut into an upper disk so that by rotating this disk only one problem can be seen at a time. The answer is covered by an adjoining flap that can be lifted for checking the answer by the child (Fig. 4).

A word may be added concerning number symbols. Number writing as well as the concrete handling of numbers is affected by functional disabilities, i.e., for example, those relating to vision or muscle coordination. Sometimes drawing and writing exercises must be introduced. Reversals such as writing 61 instead of 16, etc., can be overcome by emphasizing concretely on the abacus that 16 is one row of ten and six ones and 61 is six rows of ten and one one. The child with a space relationship impairment often has difficulty in writing his numbers correctly in vertical columns. The use of graph paper or vertically lined paper will aid such a child. Defects of perception may also play a part in the discrimination of symbols; plus, minus, and times signs can be made clearly discriminative by writing them in different colors.

Finally, we desire to point out that these methods of teaching arithmetic to mentally retarded children cannot be rigidly applied; it is necessary to consider each child's disabilities before choosing teaching methods most suitable for him. A certain procedure may bring out unexpected deficiencies with which the teacher must cope. One child, for instance, had particular difficulty with the marble-board because he could not move the marbles correctly in a vertical direction. With such a child the abacus had first to be used. With another child showing left-right confusion on the abacus the practice on the peg-board which gives opportunity for slower and larger movements was advisable. A child may have difficulties with all the mentioned devices used for learning the decimal system because of a confusion in the ordering of the various rows, i.e., for instance, mistaking 15 for 25. In such a case we introduced the even more concrete devices of counting windows in the stories of a building or the steps in several flights of stairs. One child with speech and auditory-rhythmic disabilities, though being able to handle numbers concretely, could not apply his knowledge to written problems because of the curious inability to count verbally from one upwards. Adequate exercises were necessary of combining the counting of numbers forward and backward simultaneously with the tapping or the drawing of lines in a rhythmic order.

There is one further point to be mentioned as peculiar to the remedial teaching of arithmetic. The remedial teacher is confronted with the problem of straightening out a child's distorted educational pattern. The unlearning of old habits of procedure brings the child quite frequently into a situation where he seemingly can do less than he did before. In this event the teacher must use all his skill and imagination to prevent discouragement of the child.

In conclusion, most research in the education of mentally retarded children has not been strictly enough related to the problem of organic development and its disturbance by general and individual defects. We believe that the results of systematic research programs in teaching will eventually enable the classroom teacher to understand more adequately the child's manifold difficulties; they will offer her methods which may prevent many of the so-called learning disabilities.

APPENDIX

All children of our remedial group responded favorably to these teaching methods. The following tables present data of the mental and educational level, and of the arithmetic progress of

TABLE 1

ACADEMIC ACHIEVEMENT OF SEVEN CHILDREN BEFORE REMEDIAL ARITHMETIC TRAINING

Child	C.A.	M.A.	Reading Grade	Arithmetic Grade
E. M.	12–4	8–2	3.4	2.3
C. D.	12–1	7–8	3.3	2.8
J. B.	12–2	9–4	3.4	2.6
W. G.	13–5	9–7	3.5	2.6
M. H.	11–4	6–7	3.0 *	2.5 *
M. V.	10–10	7–7	2.6 *	2.3 *
F. G.	10–10	6–1	1.8 *	2.2 *

* Metropolitan Achievement: Prim. I. A.; the first four children were tested on Standford Achievement: Prim. D.

TABLE 2

ARITHMETIC PROGRESS OF SEVEN CHILDREN AS MEASURED BY THE LOS ANGELES TESTS: FUNDAMENTALS OF ARITHMETIC

Child	Test	Add. grade	Subtr. grade	Mult. grade	Div. grade	Total	Time in mos.	Total gain grade	Ave. Gain per mo. grade
E. M.	1	2.5	2.0 (bel)	0	3.0	2.4	9.3	1.5	.15
	2	3.5	4.3	3.9	3.0	3.9			
C. D.	1	4.0	3.3	0	0	3.2	4.1	1.2	.3
	2	4.3	4.8	4.1	3.8	4.4			
J. B.	1	3.5	2.5	2.5+	0	2.9	4.2	1.4	.35
	2	4.0	4.5	4.1	3.8	4.3			
W. G.	1	2.8	2.7	0	0	2.7	5	.7	.14
	2	2.8	2.6	3.8	3.6	3.4			
M. H.	1	2.0 (bel)	0	0	0	1.5	4.6	1.3	.26
	2	3.0	2.9	0	0	2.8			
M. V.	1	2.0 (bel)	0	0	0	1.1	(4.6)	(1.5)	(.3)
	2	(2.3)*	(2.8)	(0)	(0)	(2.6)			
F. G.	1	2.0 (bel)	0	0	0	1.1	4.6	1.5	.3
	2	2.3	2.8	0	0	2.6			
						Ave. gain per mo.			.25

* M. V. was unexpectedly paroled a few days before the second examination; this child had an achievement in the remedial class of at least that of F. G.'s and possibly of M.H.'s.

a group of seven children. The remedial teaching period was approximately five months. The children had been selected as remedial arithmetic cases on recommendation by their teachers and on the basis of their achievement tests. They were taught in groups of two and three for five hours each week.

The progress in training was measured by the Los Angeles Tests: Fundamentals of Arithmetic. The tests were given immediately before and after the training period. Table 2 shows that, on the average, the children progressed one-fourth of a grade per month of training. It may be noted that the progress in academic work for our whole school population is in average approximately one-half a grade per year.[4]

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